# Section 4.2
# Mean Value Theorem

## The Mean Value Theorem for Derivatives

If y = f(x) is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Show that the function $f(x) = x^2$ satisfies the hypothesis of the Mean Value Theorem on the interval [0,2]. Then find a solution c to the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

on this interval.

**Explain why each of the following functions fails to satisfy the conditions of the Mean Value Theorem on the interval [-1,1].**

**a.)** $f(x) = \sqrt{x^2} + 1$

**b.)** $f(x) = \begin{cases} x^3 + 3, & x<1 \\ x^2 + 1, & x\geq 1 \end{cases}$

Show that $\frac{d}{dx}$ cos x has at least one zero in

[0, $2\pi$] using the MVT.

Let $f(x) = \sqrt{1 - x^2}$, $A = (-1, f(-1))$, and $B = (1, f(1))$.

Find a tangent to f in the interval $(-1,1)$ that is parallel to the secant AB.

**Suppose a trucker travels between two toll booths 13 miles apart in 15 minutes. The speed limit on the highway is 50 mph. Should the trucker be cited for speeding?**

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## Increasing Function, Decreasing Function

Let f be a function defined on an interval $I$ and let $x_1$ and $x_2$ be any two points in $I$.

1.) f increases on $I$ if $x_1 < x_2 \rightarrow f(x_1) < f(x_2)$

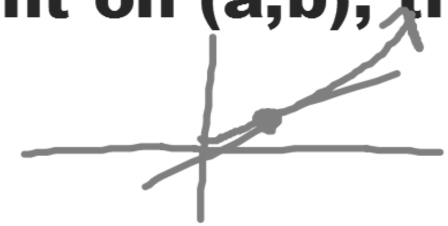
2.) f decreases on $I$ if $x_1 < x_2 \rightarrow f(x_1) > f(x_2)$

The MVT allows us to identify exactly where graphs rise and fall. Functions with positive derivatives are increasing functions. Functions with negative derivatives are decreasing functions.

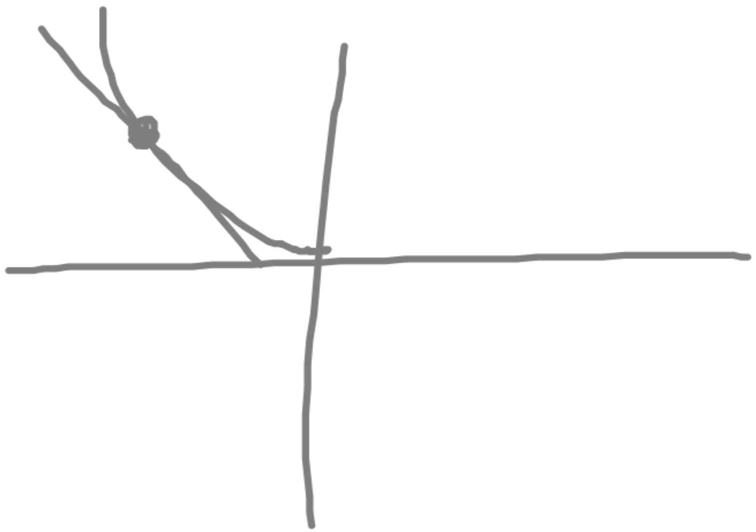
## Corrollary 1 to Increasing and Decreasing Functions

**Let f be continous on [a,b] and differentiable on (a,b)**

**1.) If f ' > 0 at each point on (a,b), then f increases on [a,b]**

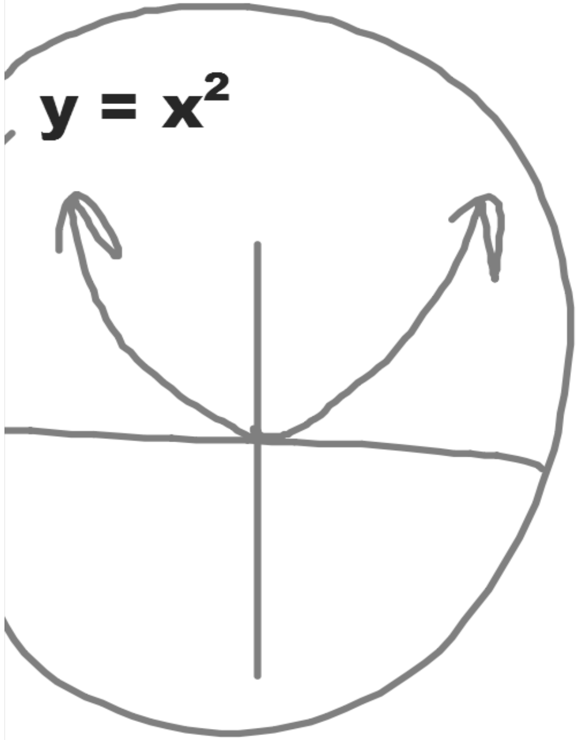


**2.) If f ' < 0 at each point of (a,b), then f decreases on [a,b]**



# Tell where the function is increasing or decreasing

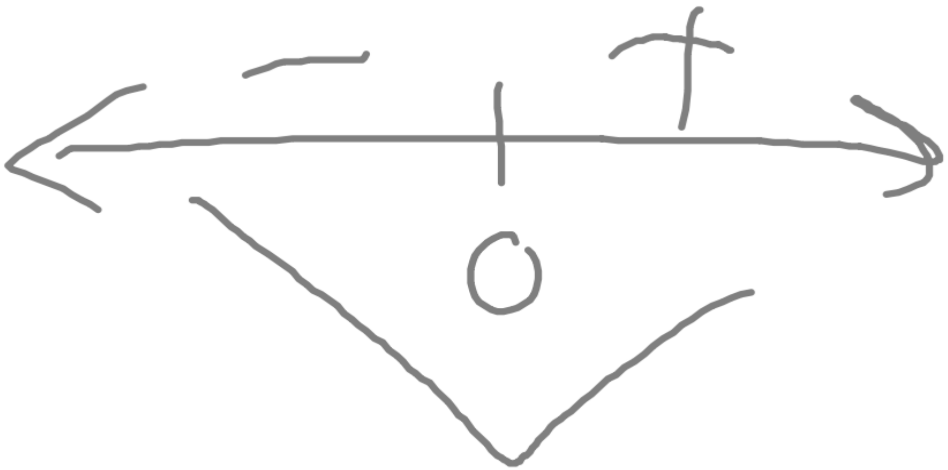
$y = x^2$



$y' = 2x$

$2x = 0$

$x = 0$



**Where is the function $f(x) = x^3 - 4x$ increasing and where is it decreasing?**

Extrema.

$f'(x) = 3x^2 - 4$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$


$-\frac{2}{\sqrt{3}}$ $\quad$ $\frac{2}{\sqrt{3}}$

local max @ $\left(-\frac{2}{\sqrt{3}}, \quad\right)$

local min @ $\left(\frac{2}{\sqrt{3}}, \quad\right)$

## Corollary 2 - Functions with f' = 0 are Constant

If $f'(x) = 0$ at each point of an interval $I$ ,

then there is a constant C for which $f(x) = C$ for all x in $I$.

## Corollary 3 - Functions with the Same Derivative Differ by a Constant

If $f'(x) = g'(x)$ at each point of an interval $I$,

then there is a constant $C$ such that

$f(x) = g(x) + C$ for all $x$ in $I$.

**Find the function f(x) whose derivative is sin x and whose graph passes through the point (0,2).**

$f(x) = -\cos x + C$

$f'(x) = \sin x$

$f(0) = 2$

$-\cos 0 + C = 2$

$-1 + c = 2$

$C = 3$

$f(x) = -\cos x + 3$

## Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all $x$ in the domain of $f$.

The process of finding an antiderivative is

antidifferentiation.

Find the velocity and position functions of a body falling freely from a height of 0 meters under each of the following sets of conditions:

a.) The acceleration is 9.8 $m/sec^2$ and the body falls from rest.

b.) The accelearation is 9.8 $m/sec^2$ and the body is propelled downward with an initial velocity of 1 m/sec.